Faculty of Science, Technology, Engineering and Mathematics
MST224 Mathematical methods

Errata list

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Handbook

Page 56	item 12	In the second displayed equation change the '$y =$' on the left hand side of the third equation to '$z =$'. The full equation should read:

$$x = x(u, v, w), \quad y = y(u, v, w), \quad z = z(u, v, w).$$

Unit 1

Page 29	Equation 45	In the Taylor expansion of cosine the signs of the terms should alternate, so that the equation should read:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots.$$

Page 37	Paragraph above Section heading	In the last sentence of the paragraph the reference should be to equation (58) rather than equation (54), so the line should read:

Using equation (58), we have

Page 44	Equation preceeding Example 4	The argument of the cosine term on the right hand side should be $x^3 + 2x$, so that the equation should read:

$$\frac{dh}{dx} = \frac{dh}{du}\frac{du}{dx} = \cos(u) \times (3x^2 + 2) = \cos(x^3 + 2x) \times (3x^2 + 2).$$

Page 55	Solution to Example 6	The unnumbered equation on line 1 should have an integral on the right hand side. The full equation should read:

$$I = \int \frac{-1}{A(x - 0)\left(x - \frac{1}{A}\right)} dx$$

Page 74	Solution to Exercise 26	In the first unnumbered equation the constant a should appear in the exponent on the right hand side so that the equation reads:

$$\frac{d}{dx}(e^{ax}) = ae^{ax}.$$

Page 80	Solution to Exercise 40(a)	In the first unnumbered equation, change from $\frac{du}{dx}du$ to $\frac{du}{dy}dy$ in the right hand side so that the equation reads:

$$\tfrac{1}{12} \int e^u \frac{du}{dy} dy.$$

Page 81	Solution to Exercise 40(c)	In the unnumbered equation on line 2, change from dx to dt in the right hand side so that the equation reads:

$$-\tfrac{1}{2} \int \sqrt{u}\, \frac{du}{dt} dt.$$

New Page 82	Solution to Exercise 42(c)	The final answer should include a constant of integration.

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Page 83	Solution to Exercise 44	In the second line of the displayed equation the variable x should have been z, so that the second line should read:

$$= \tfrac{1}{4} \left[\frac{1}{3/2} \arctan\left(\frac{z}{3/2}\right) \right]_0^{3/2}$$

Unit 2

Page 116	Example 5	In the last displayed equation there is a dx term missing. The equation should read:

$$y(x) = e^{-Ax} \left(\int e^{Ax} h(x)\, dx \right).$$

Page 120	Exercise 17	Delete the word linear from the first sentence so that the sentence reads:

Which method(s) could you use to try to solve each of the following first-order differential equations?

Page 126	Unnumbered equation below Figure 13	The right hand side of the equation should reference the numerical approximation to the solution Y_1 instead of y_1, which is the exact solution at x_1. The full equation should read:

$$Y_2 = Y_1 + h\, f(x_1, Y_1),$$

Page 135	Solution to Exercise 6(b)	When performing the substitution $u = 1 + t^2$ a factor of one half is missing outside the integral sign in one of the steps after the words "we then obtain". The corrected equation should read:

$$\int \frac{t}{1+t^2}\, dt = \tfrac{1}{2} \int \frac{1}{u} \frac{du}{dt}\, dt = \tfrac{1}{2} \int \frac{1}{u}\, du$$

Page 140	Solution to Exercise 12	In the third displayed equation the right hand side should not have been integrated, as this is the next step. So the equation should read:

$$\frac{d}{dx}(e^x y) = e^{3x}.$$

Unit 3

Page 163	Second paragraph after Equation (19)	In the last sentence before the unnumbered equations at the bottom of the page the independent variable should be x rather than t, so the end of the sentence should read,

…so substituting $y = e^{\lambda x}$ into the left-hand side of equation (19) gives,

Page 176	Equation (39)	Delete the minus sign in the exponent of the second term on the right hand side so that the equation becomes

$$x(t) = Ce^{\lambda_1 t} + De^{\lambda_2 t}.$$

New Page 176	Final paragraph before subheading	In the final sentence change the second occurrence of Γ to $-\Gamma$, so that the sentence reads:

When Γ approaches ω from above, both roots approach the value $-\Gamma$, …

Page 177	Exercise 10(c)	The question should state that the pendulum is released from rest, so the part of the question should read:

Page 177 — Exercise 10(c) — The question should state that the pendulum is released from rest, so the part of the question should read:

If the pendulum is released from rest and the initial amplitude of the oscillation is $\theta = 0.20$ (in radians), what is the amplitude at $t = 100$?

Page 192 — Second paragraph — The derivative should be with respect to the independent variable t, and so $dy/dy = b$ should be $dy/dt = b$ and $y'(t_0) = b$ should be $\dot{y}(t_0) = b$. The paragraph should read:

For a second-order differential equation, the initial conditions specify the value of the dependent variable ($y = a$) *and* the value of its derivative ($dy/dt = b$), for the *same* given value of the independent variable ($t = t_0$), and they are often written in the form $y(t_0) = a$, $\dot{y}(t_0) = b$.

Page 200 — Unnumbered equation above Equation 60 — The expression for p should contain a factor a_0 in the numerator, so that the full equation should read

$$p = \frac{-2\Gamma\omega a_0}{(\omega_0^2 - \omega^2)^2 + 4\Gamma^2\omega^2}$$

Page 201 — Paragraph preceding Figure 12 — Add the word 'approximately' in the sentence describing Figure 12(a), so that the sentence should read:

In each case, the amplitude is greatest when ω is approximately equal to the natural angular frequency ω_0 of the corresponding undriven undamped oscillator.

Unit 4

Page 5 — Exercise 1 — The question should ask for the displacement from Milton Keynes to London and not the reverse direction.

Page 34 — Highlighted box — The text above this box has been considering $m \times n$ matrices, whereas the matrices in this box are $n \times m$ matrices. For consistency the end of the sentence in the highlighted box should read:

$a_{ij} = b_{ij}$ for all $i = 1, \ldots, m$ and for all $j = 1, \ldots, n$.

Page 58 — Solution to Example 19 — In the sentence near the end of the solution that describes how to calculate the cofactor C_{14} the references should be to M_{14} rather than M_{11}, so the sentence should read:

Further, $C_{14} = (-1)^{1+4}M_{14} = -M_{14}$, where M_{14} is the determinant of the matrix obtained by crossing out the first row and fourth column of $\mathbf{A}$, i.e.

Page 74 — Solution to Exercise 32 — The two matrices on the left hand side of the equation are transposed. The first line of the solution should read:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc} \begin{bmatrix} ad-bc & -ab+ba \\ cd-dc & -cb+da \end{bmatrix}$$

Page 78 — Solution to Exercise 42(b) — There is a minor sign error when calculating the magnitude of the vector in the final line of the solution, the final line should read

$$|\mathbf{r}_1 \times \mathbf{r}_2| = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}.$$

Unit 5

Page 103	Figure 10	The figures should be labelled from left to right as (a), (b) and (c) rather than the order (a), (c) and (b) printed on the page.
Page 103	Final sentence	In the final sentence on the page the two given eigenvectors do not correspond to the values of k given. The final sentence should conclude: with $k = -2$ and $k = \frac{1}{2}$, respectively.)
New Page 108	Equation (5)	The final coefficient in the displayed equation should have subscript 3, so that the equation should read: $$\alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \alpha_3 \mathbf{v}_3 = \mathbf{0}$$
Page 110	Example 8	In the first sentence of the statement of the problem the two vectors should be column vectors, so the sentence should read: The vectors $\mathbf{v}_1 = [1 \quad 3]^T$ and $\mathbf{v}_2 = [2 \quad -1]^T$ are linearly independent.
Page 110	Example 8	The final concluding sentence is missing from the solution. The final sentence should be: So $[1 \quad 1]^T = \frac{3}{7}[1 \quad 3]^T + \frac{2}{7}[2 \quad -1]^T$.
Page 112	Solution to Example 9(b)	The vectors should be added to get the zero vector, i.e. the equation should read $$[-2 \quad 1]^T + [2 \quad -1]^T = [0 \quad 0]^T,$$
Page 113	Exercise 18	In the list of three given eigenvectors the third eigenvector should be labelled $\mathbf{v}_3$, so that the list should read: $\mathbf{v}_1 = [-1 \quad 1 \quad 1]^T$, $\mathbf{v}_2 = [1 \quad 2 \quad 1]^T$ and $\mathbf{v}_3 = [0 \quad 2 \quad 1]^T$
Page 132	Figure 18	The vector $\mathbf{v}_1$ should be pointing south east if it is representing the vector $\mathbf{i} - \mathbf{j}$.
Page 144	Solution to Exercise 8(a)	Change the first element of the vector on the right hand side of the equation from 1000 to 1100, so that the equation should read: $$\begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 1000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 1100 \\ 900 \end{bmatrix}$$
Page 146	Solution to Exercise 15	In the last sentence should have $\alpha_2 = 1$ in order to satisfy the equation, i.e. the last sentence should read: is satisfied, for example, by the values $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 0$, which are not all zero.
Page 150	Solution to Exercise 26(a)	In the penultimate displayed equation in the solution the coefficient of λ^2 should be $b - 2a$, so that the equation should read: $$a\lambda^3 + (b - 2a)\lambda^2 + (c - 2b)\lambda - 2c = 0.$$
Page 152	Solution to Exercise 32	The final line of the solution should refer to the the eigenvalue 1 rather than -1, so the final line should read: eigenvector corresponding to $\lambda = 1$ is $[1 \quad -1]^T$.

| Page 154 | Solution to Exercise 35 | For the eigenvalue $\lambda = -2$ the eigenvector found should be $k[-1 \quad 0 \quad 1]^T$, so the final sentence for this case should read: |
| | | $v = k[-1 \quad 0 \quad 1]^T$, where k is an arbitrary non-zero value. |

Unit 6

Page 193	Example 11(a)	In the hint given in part (a) the indices of the two eigenvalues should be interchanged so that eigenvalue λ_1 corresponds to eigenvector $\mathbf{v}_1$, the hint should read:
		(*Hint*: Eigenvectors of $\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$ can be shown to be
		$\mathbf{v}_1 = [1 \quad 1]^T$ with eigenvalue $\lambda_1 = -2$,
		$\mathbf{v}_2 = [1 \quad -1]^T$ with eigenvalue $\lambda_2 = -4$.)
Page 195	Highlighted box	In the sentence after the displayed equation the second mention of 'right-hand mass' should be 'left-hand mass', so that the sentence ends:
		. . . so the left-hand mass is pushed in the opposite direction by both springs attached to it.
Page 198	Text following equation 45	In the third line after equation 45 the frequency ω_2 should be ω_1, so that the equation should read:
		That is why the frequency of the in-phase normal mode, ω_1,

Unit 7

Page 43	Paragraph before green heading	The word 'is' is missing from the first sentence. The sentence should begin:
		This is the sum of two squared terms,
Page 44	Second bullet point above Procedure 1	Change 'minimum' to 'maximum' at the end of the sentence, so that the second bullet point reads:
		• When both eigenvalues are negative, $Q(x, y) < 0$, and the stationary point is a local maximum.
Page 47	Sentence above first displayed equation	Change the first mention of 'eigenvectors' to 'eigenvalues' so that the sentence begins:
		This has n real eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$
Page 47	First displayed equation	In the paragraph describing the extension from two variables (x and y) to n variables (x_1, x_2, ... x_n) the quadratic function $Q(x, y)$ will become a function of the n variables, so the displayed equation should read:
		$Q(x_1, x_2, \ldots, x_n) = c_1^2 \lambda_1 + c_2^2 \lambda_2 + \cdots + c_n^2 \lambda_n$

Unit 8

| Page 105 | Box containing Equation (33) | In the sentence below Equation 33 the last scale factor in the list should be h_w, so the sentence reads: |
| | | where h_u, h_v and h_w are appropriate scale factors. |

| Page 112 | Box containing Equation (46) | In the final sentence the range for the variable v should be from v_1 to v_2 and not from v_1 to u_2 as printed, so that the middle of the final sentence should read:

... and $v_1 \leq v \leq v_2$ are chosen to cover... |

Unit 9

Page 141	Main text, describing Figure 11	The word 'as' is missing from the final sentence, which should read: As you would expect, all the arrows point towards the centre of the star, and their length increases as they get closer to the star.
Page 147	Main text, 7 lines from bottom	Change v_y to v_ϕ so that the sentence reads: An expression for v_ϕ can be found in a similar way.
Page 162	Sentence following Equation (39)	Change $V(x, y, x)$ to $V(x, y, z)$ so that the sentence begins: When this operator acts on a scalar field $V(x, y, z)$
Page 193	Solution to Eexercise 23(a)	In the first sentence change '$F_\phi = 0$ and $F_z = 0$' to '$F_\theta = 0$ and $F_\phi = 0$' so that the second line of text reads: $F_r = 4r^3$, $F_\theta = 0$ and $F_\phi = 0$, we get

Unit 10

Page 209	Exercise 2	Change 'parabolic' to 'parametric' in the first sentence, so that the sentence reads: One turn of a helical path has the parametric representation
New Page 214	Caption of Figure 15	The date of birth of Albert Einstein is 14 March 1879, so the caption to Figure 15 should read: Albert Einstein (1879–1955)
Page 236	Solution to Example 10	When working out the flux over the surface one of the intermediate lines has an R^2 factor instead of R^3. The correct line should read: $\mathbf{F} \cdot \mathbf{J} = R^3 \sin^2\theta \left(\cos\phi\,\mathbf{i} + \sin\phi\,\mathbf{j}\right) \cdot \mathbf{e}_r$.
Page 252	Exercise 22	The last coordinate in the list of vertices of the given rectangle should be $(0, 1)$, so the final line should read: ...at $(0, 0)$, $(2, 0)$, $(2, 1)$, $(0, 1)$, traversed in that order, and returning to $(0, 0)$.

Unit 11

| Page 46 | Exercise 20 | The range of definition of the function f should be from $-1/4$ rather than $1/4$, so that the definition should read:

$f(x) = x$ for $-\frac{1}{4} < x \leq \frac{3}{4}$,
$f(x + 1) = f(x)$, |

Page 55	Example 15	There are two sign errors in the denominators of fractions at the end of the solution. The final two lines should read:

$$x(t) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{i(-1)^n}{n} \frac{(\omega_0^2 - n^2\omega^2) - 2\Gamma i n\omega}{(\omega_0^2 - n^2\omega^2)^2 - (2\Gamma i n\omega)^2} e^{int}$$

$$= \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n} \frac{2\Gamma n\omega + i(\omega_0^2 - n^2\omega^2)}{(\omega_0^2 - n^2\omega^2)^2 + 4\Gamma^2 n^2\omega^2} \left(\cos(nt) + i\sin(nt)\right).$$

Page 60	Margin note	The exponential terms should be e^{ix} and e^{-ix} rather than e^x and e^{-x}, so that the margin note should read:

Recall that $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$.

Page 77	Solution to Exercise 21	In the introductory paragraph of the solution the function $s'(t)$ (mentioned twice) should be simply $s(t)$, so that the paragraph should start:

The function $c(t)$ is continuous, as can be seen from its graph, and $s(t)$ is *minus* its derivative. The Fourier series for $s(t)$ is denoted by $S(t) \ldots$

Unit 12

Page 101	Displayed equation above Figure 9	In the definition of the odd extension f_{odd} the definition for $-L < x < 0$ should be $-f(-x)$, so that the equation should read

$$f_{\text{odd}}(x) = \begin{cases} f(x) & \text{for } 0 \leq x \leq L, \\ -f(-x) & \text{for } -L < x < 0, \end{cases}$$

$$f_{\text{odd}}(x + 2L) = f_{\text{odd}}(x).$$

New Page 109	Figure 15	At the top of the figure the two formulas for the number of particles entering and leaving should include a factor ΔA, so the top of the diagram should read:

$$J_x(x, t)\, \Delta A\, \delta t \qquad\qquad J_x(x + \Delta x, t)\, \Delta A\, \delta t$$

particles enter $\longrightarrow$ $\longrightarrow$ particles leave

Page 113	Example 2	The range of validity of the partial differential equation should be for x less than L rather than x less than 1 as stated, so the partial differential equation should read:

$$\frac{\partial^2 c}{\partial x^2} = \frac{1}{D} \frac{\partial c}{\partial t} \quad (0 < x < L,\ t > 0)$$

Page 119	Figure 16	The horizontal axis should be labelled x rather than t.
Page 120	Step 5	In the bold heading the function should be $T_n(t)$ rather than $T_n(x)$, so the heading should read:

Step 5   Find the functions $T_n(t)$

Page 121	Step 6	In the opening sentence the function should be $X_n(x)$ rather than $X_n(t)$, so the sentence should start:

By combining $X_n(x)$ and $T_n(t)$,

Page 131   Unnumbered   In the unnumbered displayed equation the integration should be with
            displayed   respect to s rather than u and the lower limit of integration should
            equation   be minus infinity, so that the equation should read

$$c(x,t) = \frac{1}{\sqrt{4\pi Dt}} \int_{-\infty}^{\infty} \exp\left[-\frac{(x-s)^2}{4Dt}\right] c_0(s)\,ds$$

Unit 13

Page 187   Figure 26   The direction arrow on the separatrix just below the point labelled
                          C should point away from the equilibrium point at $(\pi, 0)$.